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THESIS

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ABSTRACT

Methods for approximating the system hazard function are developed for systems which have constant component failure rates. The approximations are applicable to systems which are "highly reliable," e.g., all component reliabilities greater than 0.9 and system reliability greater than 0.95.

Three approximations are developed. The first-order approximation is based on the system cuts of least size (smallest cuts). The fix-up approximation is based on the minimal cuts of the system and is an extension of the methods used in NAVWEPS OD 29304. The second-order approximation is a more accurate extension of the first-order approximation.

The advantages peculiar to each of these approximations are:

(1) first-order: easy to calculate,

(2) fix-up: never leads to an overestimate of system reliability,

(3) second-order: relatively more accurate.

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I. INTRODUCTION

This thesis deals with probabilistic approximations for system reliability as opposed to statistical estimators of component or system failure rates. The problem addressed herein is that of approximating the probability of failure as a function of time for a system whose components have known (or estimated) constant failure rates.

It has been shown [1] that a system of constant failure rate components will not have a constant system failure rate unless all components are in series. When all components are in series the system failure rate is the sum of the component failure rates, a simple calculation requiring no approximations. When all components are not in series the system failure rate is a function of time; usually it is a rather complicated function. In the latter case simple approximations have great practical usefulness.

The function approximated in this thesis is the system hazard function. The hazard function is a very convenient device when a system consists of a series of subsystems, and some of the subsystems have components in parallel. The system hazard function is simply the sum of the subsystem hazard functions, and the system reliability is the exponential of the negative of the system hazard function. The only difficulty in determining system reliability, then, is determining the hazard function for those subsystems which do not

have all components in series. It is to this end that simple approximate methods for determining the hazard function are developed in this paper.

The system hazard function can be formulated as a function of the component failure rates and of time (mission length). Bounds on the value of the hazard function for any mission length are developed in [2]. The approximations developed in this paper are useful for systems of variable mission length, however, the accuracy of the approximations is generally acceptable only in the range of mission lengths for which system reliability is "high" (e.g., component reliabilities at least 0.9 and system reliability at least 0.95).

Three methods of approximating the system hazard function are developed in this paper:

(1) first-order approximation

(2) fix-up approximation

(3) second-order approximation.

The first-order approximation has the form $a(\lambda t)^b$ when component failure rates are all equal. When component failure rates are not all equal the first-order approximation has the form $\sum\Pi(\lambda_i t)$. The fix-up and the second-order approximations are made up of terms of the same form as the first-order approximation. The fix-up approximation is an extension of the methods used in [3]. When the methods of [3] can be applied unambiguously they give the same result as the fix-up approximation.

None of these approximations requires the complete computation of the system reliability function. This is of definite practical advantage in complex systems.

II. DEFINITIONS AND EXAMPLES

A convenient vehicle for illustrating definitions is the 2-out-of-3 system. A 2-out-of-3 system consists of three components: it functions if any two components function or all three components function. If less than two components function, the system fails.

A. SYSTEM STRUCTURE

All components and systems are considered to be two-state devices in the sense that they either "function" or "fail." We use the symbol x_i to represent the state of the i-th component:

$$x_i = 0 \text{ if the i-th component fails,}$$
$$= 1 \text{ if the i-th component functions.}$$

1. Condition Vector

In a system consisting of n components the vector $x = (x_1,\ldots,x_n)$ represents the state of all the components in the system. We say x describes the "condition" of the system. Superscripts are used to distinguish different conditions of the system, e.g., x^1 , x^2 .

The possible conditions of a 2-out-of-3 system are shown in Figure 1.

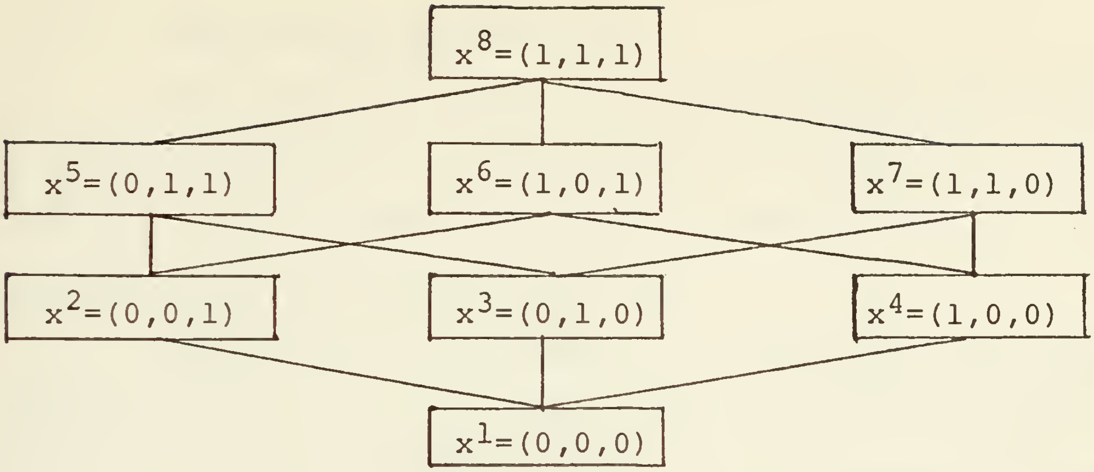

Figure 1. Conditions of a 2-out-of-3 System.

2. Structure Function

The structure function $\phi(x)$ indicates the state of the system (function or fail) when it is in the condition described by x:

$$\begin{aligned} \phi(x) &= 0 \quad \text{if the system fails in condition } x, \\ &= 1 \quad \text{if the system functions in condition } x. \end{aligned}$$

Considering the conditions of a 2-out-of-3 system shown in Fig. 1, we see for example that:

$\phi(x^2) = 0$ (only component No. 3 functions; system fails),

$\phi(x^6) = 1$ (components Nos. 1 and 3 function; system functions).

3. Cut

The approximations developed in this thesis are closely related to those conditions of the system in which it fails. We define a cut as any condition in which the system fails, i.e., condition x is a cut if $\phi(x)=0$.

The 2-out-of-3 system has four cuts: $x^1=(0,0,0)$, $x^2=(0,0,1)$, $x^3=(0,1,0)$ and $x^4=(1,0,0)$. The system fails if it is in any of these four conditions.

4. Cut Size and System Width

The size of a cut x, s(x), is defined as the number of failed components in the condition described by x. For example, in Fig. 1 the size of $x^1=(0,0,0)$ is $s(x^1) = 3$, and the size of $x^2=(0,0,1)$ is $s(x^2) = 2$.

The width of the system, m, is defined as the smallest size of any cut, i.e., the least number of failed components that can cause the system to fail. In a 2-out-of-3 system at least two components must fail in order that the system fails, so the system width is $m = 2$.

5. Failed Component Set

For any condition x the failed component set 0(x) is defined as the set of indices of the components that are failed when the system is in the condition described by x. In Fig. 1, $0(x^4) = \{2,3\}$ and $0(x^5) = \{1\}$.

6. Minimal Cut

The concept of a "minimal" cut is important in the approximations developed in this paper. A cut x is a minimal cut if it has the following property: the "repair" of any failed component in the cut causes the system to function. In the 2-out-of-3 system the conditions x^2 , x^3 and x^4 are minimal cuts; x^1 is not a minimal cut. In general, a cut x^A is said to "contain" a cut x^B if $0(x^B)$ is a proper subset of $0(x^A)$. Consequently, a minimal cut contains no other cuts. In Fig. 1, $0(x^2) = \{1,2\}$ is a proper subset of $0(x^1) = \{1,2,3\}$; x^2 is a cut, therefore, x^1 is not a minimal cut. Note that if the size of a cut equals the system width, then the cut is minimal.

7. Sets of Cuts

We define K as the set of all cuts of the system and K_s as the set of all cuts of size s. C_s is defined as the number of cuts of size s. For the 2-out-of-3 system we have:

$K = \{x^1, x^2, x^3, x^4\}$

K_1 = the empty set $\qquad C_1 = 0$

$K_2 = \{x^2, x^3, x^4\}$ $\qquad C_2 = 3$

$K_3 = \{x^1\}$ $\qquad C_3 = 1.$

B. PROBABILISTIC RELIABILITY FUNCTIONS

So far the probabilistic and time dependent characteristics of the component and system states have been suppressed. It is assumed, however, that only probabilistic statements can be made about the state of the components and the system, and that these statements have time as an independent variable.

A system consisting of two components in series provides a convenient example for illustrating the definitions of this section. Such a system functions only if both components function; if either component or both components fail, then the system fails.

1. Reliability Function

In the example of a two-component series system an important question is, "Will component No. 1 still be functioning at time t?" The state of component No. 1, x_1, is considered a probabilistic function of time, i.e., $x_1 = X_1(t)$, a random variable. So a more meaningful form of the above question is, "What is the probability $x_1 = 1$ at time t?" The

answer to this question is called the component reliability function, $\overline{F}_1(t)$.

In general, for any i-th component the component reliability function $\overline{F}_i(t)$ is defined as

$$\overline{F}_i(t) \equiv \Pr(\text{i-th component is functioning at time t})$$

$$= \Pr(X_i(t) = 1).$$

The form of the component reliability function depends on the reliability characteristics of the component. In the two-component series system example the "exponential lifetime" form is used, i.e.,

$$\overline{F}_1(t) = e^{-\lambda_2 t}$$

$$\overline{F}_2(t) = e^{-\lambda_2 t}$$

where λ_1 and λ_2 are positive constants.

Turning our attention to the system we might ask, "What is the probability that the system is still functioning at time t?" The answer to this question is called the system reliability function $\overline{F}(t)$, i.e.,

$$\overline{F}(t) \equiv \Pr(\text{system is functioning at time t}).$$

In the two-component series system the system reliability function is

$$\overline{F}(t) = \Pr(\text{both components are functioning at time t})$$

$$= \overline{F}_1(t)\ \overline{F}_2(t)$$

$$= e^{-\lambda_1 t}\ e^{-\lambda_2 t}$$

$$= e^{-(\lambda_1 + \lambda_2)t}.$$

2. System Reliability in Terms of Cuts

There are various methods for computing system reliability. The following method, although it appears rather cumbersome, proves useful in developing hazard function approximations.

For any condition of the system, x, define p(x) as the probability that the system is in condition x at time t. It is assumed that the failure times of the components in the system are independent random variables. Consequently, p(x) equals the product of the probabilities that each component is in the state described by condition x.

The four possible conditions of a two-component series system are shown in Fig. 2.

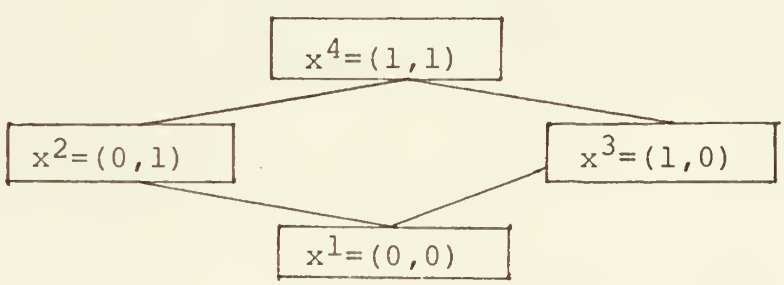

Figure 2. Conditions of a Two-Component Series System.

In condition x^2 the component states are $x_1=0$ (No. 1 component has failed) and $x_2=1$ (No. 2 component is functioning).

So, $p(x^2)$ = Pr(No. 1 comp. has failed and No. 2 comp. is functioning).

= Pr(No. 1 comp. has failed) Pr(No. 2 comp. is functioning.

But, Pr(No. 1 comp. has failed) = 1 - Pr(No. 1 comp. is functioning)

$$= 1 - \overline{F}_1(t)$$

$$= 1 - e^{-\lambda_1 t} .$$

Also, Pr(No. 2 comp. is functioning) = $e^{-\lambda_2 t}$.

Thus, $p(x) = (1-e^{-\lambda_1 t})e^{-\lambda_2 t}$.

The probability that the system has failed equals the probability that it is in a condition which is a cut. The conditions of a system are mutually exclusive (if the system is in condition x^A it is not in any other condition x^B). Consequently, we can sum the probabilities of the conditions that are cuts to get the probability that the system has failed.

The two-component series system has three conditions which are cuts, i.e.,

$$K = \{x^1, x^2, x^3\}.$$

The probability this system has failed is the sum over the conditions in K of the probabilities of being in those conditions, i.e.,

$$\begin{aligned} \text{Pr(system has failed)} &= \sum_{x \varepsilon K} p(x) \\ &= p(x^1) + p(x^2) + p(x^3). \end{aligned}$$

Using the component reliability functions, $\overline{F}_1(t)$ and $\overline{F}_2(t)$,

we have: $$\begin{aligned} p(x^1) &= (1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t}) \\ p(x^2) &= (1-e^{-\lambda_1 t})e^{-\lambda_2 t} \\ p(x^3) &= e^{-\lambda_1 t}(1-e^{-\lambda_2 t}). \end{aligned}$$

Summing these,

$$\text{Pr(system has failed)} = 1 - e^{-(\lambda_1+\lambda_2)t} .$$

Thus, $\bar{F}(t)$ = Pr(system is functioning)

$$= 1 - \Pr(\text{system has failed})$$

$$= e^{-(\lambda_1+\lambda_2)t} .$$

The foregoing method of calculating system reliability in terms of cuts has three significant characteristics:

(1) It can be used for any system.

(2) It requires only a knowledge of the conditions which are cuts.

(3) Cuts which have relatively small probability can be ignored in order to simplify the calculation of approximate system reliability.

3. Hazard Function

For the i-th component the component hazard function $R_i(t)$ is defined by

$$R_i(t) \equiv -\log \bar{F}_i(t).$$

The system hazard function R(t) is similarly defined by

$$R(t) \equiv -\log \bar{F}(t).$$

In the two-component series system we have

$$R_1(t) = -\log e^{-\lambda_1 t} = \lambda_1 t,$$

$$R_2(t) = -\log e^{-\lambda_2 t} = \lambda_2 t,$$

$$R(t) = -\log e^{-(\lambda_1+\lambda_2)t} = (\lambda_1+\lambda_2)t.$$

Note that in this series system the system hazard function is the sum of the component hazard functions, i.e.,

$$R(t) = R_1(t) + R_2(t).$$

The hazard function is very convenient to use when dealing with systems which can be represented as a series of subsystems. Each subsystem hazard function can be calculated or approximated, and the system hazard function is then simply the sum of the subsystem hazard functions. The system reliability is the negative exponential of the system hazard function, i.e., $\bar{F}(t) = e^{-R(t)}$.

4. Failure Rate

For the i-th component the component failure rate $r_i(t)$ is defined by

$$r_i(t) \equiv \frac{d}{dt} R_i(t).$$

The system failure rate r(t) is similarly defined by

$$r(t) \equiv \frac{d}{dt} R(t).$$

(These definitions agree with the classical definition of failure rate in terms of the probability density function and the reliability function.)

In the two-component series system we have

$$r_1(t) = \frac{d}{dt} R_1(t) = \frac{d}{dt} \lambda_1 t = \lambda_1 \ ,$$

$$r_2(t) = \frac{d}{dt} R_2(t) = \frac{d}{dt} \lambda_1 t = \lambda_1 \ ,$$

$$r(t) = \frac{d}{dt} R(t) = \frac{d}{dt} (\lambda_1 + \lambda_2) t = \lambda_1 + \lambda_2 \ .$$

Note that the component failure rates, λ_1 and λ_2, are constant. An exponential lifetime ($\bar{F}_i(t) = e^{-\lambda_i t}$) is equivalent to a constant failure rate ($r_i(t) = \lambda_i$).

In the systems considered in this thesis all components have constant failure rates.

III. FIRST-ORDER APPROXIMATIONS

The problem to be dealt with here is that of determining the hazard function for a large system in which all components have constant failure rates. It is assumed that the large system consists of a series of subsystems. Those subsystems which consist of a single component or a series of components may be treated separately; the hazard function for this group is simply the sum of the component hazard functions. An exact formulation of the hazard function for the more complex subsystems is often difficult. The approximations developed in this section provide a relatively simple method for approximating the hazard function for these complex subsystems.

(The "systems" referred to in this section should be considered as subsystems within a larger system. These "systems" always have more than one component and are more complex than a simple series of components.)

Initially, consider first-order approximations for systems in which all failure rates are equal to some constant $\lambda (>0)$. Once the method is illustrated in this special case, the generalization to systems in which component failure rates are not all equal is rather straightforward.

The following example shows a direct technique for deriving a first-order approximation to the hazard function.

Example 3-1. Consider a system consisting of three components each having constant failure rate λ. The system functions if and only if two or more of its components function (2-out-of-3 system).

The component reliability functions are

$$\bar{F}_i(t) = e^{-\lambda t} \qquad i = 1,2,3.$$

The system reliability function is

$$\bar{F}(t) = 3e^{-2\lambda t} - 2e^{-3\lambda t}.$$

Expand the exponentials in the system reliability function using $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$.

We then have

$$\begin{aligned}\bar{F}(t) &= 3(1 - 2\lambda t + \frac{4(\lambda t)^2}{2!} - \frac{8(\lambda t)^3}{3!} + \ldots)\\ &\quad -2(1 - 3\lambda t + \frac{9(\lambda t)^2}{2!} - \frac{27(\lambda t)^3}{3!} + \ldots)\\ &= 1 \qquad -3(\lambda t)^2 + 5(\lambda t)^3 + \ldots .\end{aligned}$$

We can now derive the hazard function

$$R(t) = -\log \bar{F}(t)$$

by using the expansion of the logarithm function about 1:

$$-\log(1-x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots .$$

Thus, $R(t) = -\log(1 - 3(\lambda t)^2 + 5(\lambda t)^3 - \ldots)$

$$= 3(\lambda t)^2 - 5(\lambda t)^3 + \ldots .$$

The first-order approximation, then, is

$$R^1(t) = 3(\lambda t)^2.$$

The direct method has two drawbacks. First, an explicit formulation of the system reliability function is required. Second, the exponentials must be expanded and the coefficients of the various powers of λt must be collected.

The indirect method of determining the first-order approximation for the hazard function is based on the following line of reasoning:

(1) Recall that K is the set of all cuts of the system, and that the elements of K are mutually exclusive. Also, because of the assumption that component lifetimes are independent, the probability that the system is in the condition described by a cut equals the product of the probabilities that each component is in the state described by the cut.

(2) The reliability function can be formulated as

$$\begin{aligned}\Pr(\text{system is functioning}) &= 1 - \Pr(\text{system has failed})\\ &= 1 - \sum_{x \varepsilon K} \Pr(\text{system in condition } x).\end{aligned}$$

(3) For any cut x of size s (size = no. failed comp.),

$$\begin{aligned}p(x) &= \Pr(\text{system in condition } x)\\ &= (e^{-\lambda t})^{n-s}(1-e^{-\lambda t})^{s}.\end{aligned}$$

Consider the value of $p(x)$ for small values of t,

$$\lim_{t\to 0} p(x) = \lim_{t\to 0} (e^{-\lambda t})^{n-s}\left(\frac{1-e^{-\lambda t}}{t}\right)^{s} t^{s-r} \qquad (r\leq s).$$

But, $\lim_{t\to 0} (e^{-\lambda t})^{n-s} = 1,$

$$\lim_{t\to 0}\left(\frac{1-e^{-\lambda t}}{t}\right)^{s} = \lambda^{s},$$

$$\begin{aligned}\lim_{t\to 0} t^{s-r} &= 0 \quad \text{if } r<s,\\ &= 1 \quad \text{if } r=s.\end{aligned}$$

Thus, when x is a cut of size s,

$$\begin{aligned}\lim_{t\to 0} \frac{p(x)}{t^{r}} &= 0 \text{ for } r<s,\\ &= \lambda^{s} \text{ for } r=s.\end{aligned}$$

For small values of t we have the approximation

$$p(x) \doteq (\lambda t)^s.$$

(4) Recall C_s is the number of cuts of size s, and m is the smallest value of s such that $C_s > 0$. (m ≡ system width.)

$$\overline{F}(t) = \Pr(\text{system is functioning})$$

$$= 1 - \sum_{x \in K} p(x)$$

$$\doteq 1 - \sum_{s=m}^{n} C_s (\lambda t)^s$$

$$\doteq 1 - C_m (\lambda t)^m.$$

(5) The first-order approximation for the system hazard function is derived from the foregoing approximation for the system reliability function.

$$R(t) = -\log \overline{F}(t)$$

$$\doteq -\log(1 - C_m (\lambda t)^m)$$

$\doteq C_m (\lambda t)^m \equiv R^1(t)$ where $R^1(t)$ is the first-order approximation for the system hazard function.

In summary, when all component failure rates are equal the first-order approximation for the system hazard function can be determined in three steps:

a. Determine the system width, m.

b. Determine the number of cuts of size m, C_m.

c. Calculate $R^1(t) = C_m (\lambda t)^m$.

Example 3-2. Apply the foregoing method to the 2-out-of-3 system.

a. At least two components must fail for the system to fail, so the system width is two (m=2).

b. There are three cuts of size two, so $C_m = 3$.

c. Consequently, $R^1(t) = C_m(\lambda t)^m = 3(\lambda t)^2$.

When the component failure rates are not all equal the first-order approximation for the hazard function is derived by the same line of reasoning used in (1) through (5) above. The essential difference is in the form of p(x).

Let x be a cut of size s. Recall $0(x) \equiv \{i:x_i=0\}$, and $K_s \equiv$ {cuts of size s}. By the assumption of independent component lifetimes,

$$p(x) = \prod_{i \notin 0(x)} e^{-\lambda_i t} \prod_{i \in 0(x)} (1-e^{-\lambda_i t}).$$

There are s elements in 0(x), so

$$\frac{p(x)}{t^r} = \prod_{i \notin 0(x)} e^{-\lambda_i t} \prod_{i \in 0(x)} \left(\frac{1-e^{-\lambda_i t}}{t}\right) t^{s-r} \qquad (r \leq s),$$

$$\text{and } \lim_{t \to 0} \frac{p(x)}{t^r} = 0 \qquad r<s,$$

$$= \prod_{i \in 0(x)} \lambda_i \qquad r=s.$$

Thus, for small t, $p(x) \doteq t^s \prod_{i \in 0(x)} \lambda_i$.

The system reliability function is approximated by

$$\bar{F}(t) \doteq 1 - \sum_{x \in K_m} p(x)$$

$$\doteq 1 - \sum_{x \in K_m} t^m \prod_{i \in 0(x)} \lambda_i .$$

The first-order approximation for the system hazard function follows from

$$R(t) = -\log \bar{F}(t)$$
$$\doteq \sum_{x \varepsilon K_m} t^m \prod_{i \varepsilon 0(x)} \lambda_i = R^1(t).$$

In summary, when the component failure rates are not all equal the first-order approximation for the system hazard function can be determined in three steps:

a. Determine the system width, m.

b. For each cut x in K_m calculate the product

$$\prod_{i \varepsilon 0(x)} \lambda_i t = t^m \prod_{i \varepsilon 0(x)} \lambda_i .$$

c. The first-order approximation $R^1(t)$ is the sum of these products over all x in K_m, i.e.,

$$R^1(t) = t^m \sum_{x \varepsilon K_m} \prod_{i \varepsilon 0(x)} \lambda_i .$$

The following example illustrates the method.

Example 3-3. Consider a 2-out-of-3 system with component failure rates λ_1, λ_2 and λ_3. Recall that the cuts of the system are:

$x^1 = (0,0,0)$,

$x^2 = (0,0,1)$,

$x^3 = (0,1,0)$,

and $x^4 = (1,0,0)$.

Thus, $K_2 = \{x^2, x^3, x^4\}$, and $K_3 = \{x^1\}$.

The width of the system is $m = 2$.

For each cut x in K_2 form the product $\prod_{i \varepsilon 0(x)} \lambda_i t$:

x^2: $\lambda_1 \lambda_2 t^2$

x^3: $\lambda_1 \lambda_3 t^2$

x^4: $\lambda_2 \lambda_3 t^2$.

Summing these products,

$$R^1(t) = t^m \sum_{x \varepsilon K_m} \prod_{i \varepsilon 0(x)} \lambda_i$$

$$= t^2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3).$$

If the component failure rates had been equal, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$, then the above answer would reduce to

$$R^1(t) = t^2(3\lambda^2) = 3(\lambda t)^2.$$

The first-order approximation for the system hazard function contains three approximations:

(1) For any i-th component with constant failure rate λ_i,

$$1 - e^{-\lambda_i t} \doteq \lambda_i t.$$

(2) $\Pr(\text{system has failed}) \doteq \sum_{x \varepsilon K_m} p(x).$

(3) $-\log(1-\Pr(\text{system has failed})) \doteq \Pr(\text{system has failed}).$

Approximation (1) has an error of less than 5% when $\lambda_i t \leq 0.10$. This corresponds to a component reliability of 0.9 or greater.

The error in approximation (2) is directly related to the probability that the system fails due to a cut of size greater than system width m. The accuracy of approximation (2) suffers when relatively unreliable components are not in the failed component set for any cut of size m. When system reliability is greater than 0.95 the cuts of size m usually account for enough of the probability of system failure to make approximation (2) acceptable.

Approximation (3) has an error of less than 5% when system reliability is greater than 0.90.

As a rule of thumb we say that the first-order approximation for the system hazard function is usually acceptable when all component reliabilities are greater than 0.9 and the system reliability is greater than 0.95.

IV. FIX-UP AND SECOND-ORDER APPROXIMATIONS

In this section we develop methods for improving upon the first-order approximation for the system hazard function. The first method developed, the fix-up approximation, is not always a more accurate approximation than the first-order approximation, but it always errs on the "safe side." The other method developed, the second-order approximation, is always at least as accurate as the first-order approximation, in most cases it is significantly more accurate.

A. THE FIX-UP APPROXIMATION

The first-order approximation of the system hazard function may not be acceptable if comparatively "weak" components do not appear in any of the cuts of size m (m = system width). The following example illustrates this situation and provides a heuristic approach to the fix-up approximation.

Example 4-1. Consider the hydraulic pump system shown in Fig. 3.

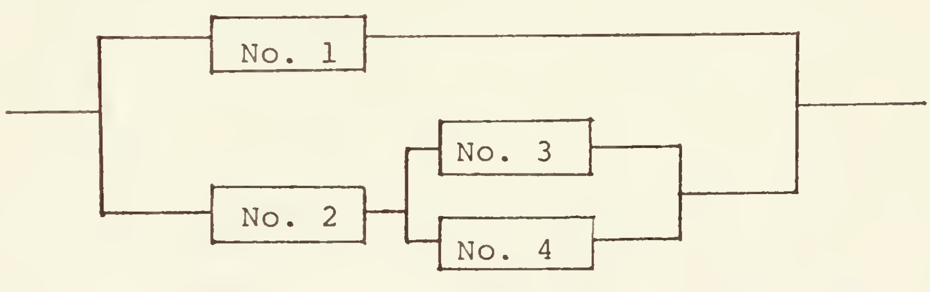

Figure 3. Hydraulic Pump System - Physical Arrangement.

The upper branch functions if No. 1 functions. The lower branch functions if No. 2 functions and either No. 3 or No. 4 functions. The system functions if either the upper or lower branch functions.

The cut representation of the system is shown in Fig. 4.

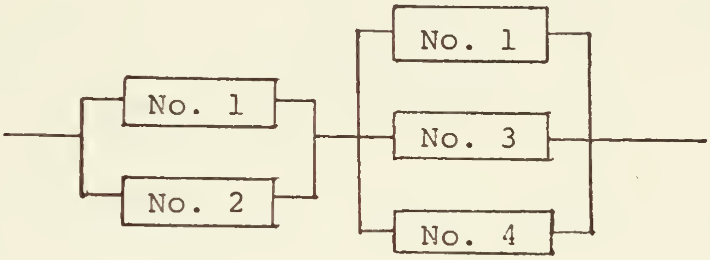

Figure 4. Hydraulic Pump System - Cut Representation.

The first-order approximation for the system hazard function is $R^1(t) = \lambda_1\lambda_2 t^2$. This completely ignores the failure rates of Nos. 3 and 4. If Nos. 3 and 4 are considerably less reliable than Nos. 1 and 2, the accuracy of the first-order approximation may not be acceptable. An obvious "fix-up" would be to get a first-order approximation based on the cut in which Nos. 1, 3 and 4 are failed, and add this to the previous approximation, i.e.,

$$R^F(t) = R^1(t) + \lambda_1\lambda_3\lambda_4 t^3$$

$$= \lambda_1\lambda_2 t^2 + \lambda_1\lambda_3\lambda_1 t^3.$$

The "fix-up" approximation for the system hazard function, $R^F(t)$, shown in Example 4-1 is sometimes a simple method for improving the accuracy of the first-order approximation. It can be derived for any system in the following manner:

a. Identify all minimal cuts of the system.

b. For any minimal cut x, let s(x) = size of x.

c. Let K_M = {all minimal cuts}.

d. Then, $R^F(t) = \sum_{x \varepsilon K_M} t^{s(x)} \prod_{i \varepsilon 0(x)} \lambda_i$.

For illustration, apply these steps to Example 4-1:

a. There are two minimal cuts,

$x^1 = (0,0,1,1)$,

$x^2 = (0,1,0,0)$.

b. $s(x^1) = 2$, and $s(x^2) = 3$.

c. $K_M = \{x^1, x^2\}$.

d. $$R^F(t) = \sum_{x \varepsilon K_M} t^{s(x)} \prod_{i \varepsilon 0(x)} \lambda_i$$

$$= t^2(\lambda_1\lambda_2) + t^3(\lambda_1\lambda_3\lambda_4)$$

$$= \lambda_1\lambda_2 t^2 + \lambda_1\lambda_3\lambda_4 t^3.$$

The rationale behind the fix-up approximation can be summarized in the following manner. We treat each minimal cut x as if the system width were s(x). We then calculate the "contribution" of the minimal cut x to the first-order approximation for the system hazard function, $t^{s(x)} \prod_{i \varepsilon 0(x)} \lambda_i$. These contributions are summed over all minimal cuts, and the result is the fix-up approximation.

The fix-up approximation for the system hazard function is always greater than the actual system hazard function, i.e., $R^F(t) > R(t)$.

Recall that all cuts of size m are minimal cuts, so K_m is a subset of K_M. Consequently,

$$R^F(t) = \sum_{x \varepsilon K_M} t^{s(x)} \prod_{i \varepsilon 0(x)} \lambda_i \geq \sum_{x \varepsilon K_m} t^m \prod_{i \varepsilon 0(x)} \lambda_i = R^1(t).$$

In view of these inequalities the fix-up approximation is less accurate than the first-order approximation whenever the first-order approximation is "pessimistic" i.e., whenever $R^1(t) > R(t)$.

It is important to note that the fix-up approximation does provide an upper bound on the system hazard function and, therefore, it leads to a lower bound on system reliability. If errors of "optimism" are to be avoided, the fix-up approximation has the virtue that any errors will be errors of "pessimism."

The methods for approximating the system hazard function shown in Ref. 3 give results which are equivalent to the fix-up approximation. (Unfortunately, these methods cannot be applied unambiguously to some systems, e.g., the 2-out-of-3 system.)

B. THE SECOND-ORDER APPROXIMATION

When component failure rates are constant it is always possible to express the system hazard function as a power series:

$$R(t) = \sum_{j=0}^{\infty} a_j t^j$$

where a_j is a function of the component failure rates. It was

shown in developing the first-order approximation that $a_j = 0$ when j is less than the system width m. Consequently, the power series expansion of R(t) can be written:

$$R(t) = \sum_{j=m}^{\infty} a_j t^j \ .$$

The first-order approximation was simply the first term in the power series expansion of R(t), i.e.,

$$R^1(t) = a_m t^m$$

where $a_m = \sum_{x \varepsilon K_m} \prod_{i \varepsilon 0(x)} \lambda_i$.

The second-order approximation for the system hazard function, $R^2(t)$, consists of the first two non-zero terms of the power series expansion of R(t), i.e.,

$$R^2(t) = a_m t^m + a_{m+1} t^{m+1} \ .$$

One method for calculating the second-order approximation is the direct approach (see Example 3-1). To illustrate this method consider the system in Example 4-1.

The system reliability function is

$$\overline{F}(t) = e^{-\lambda_1 t} + (1-e^{-\lambda_1 t})e^{-\lambda_2 t}(e^{-\lambda_3 t} + e^{-\lambda_4 t} - e^{-(\lambda_3+\lambda_4)t}).$$

The direct approach consists of three steps:

a. Expand the exponentials in $\overline{F}(t)$ using

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots \ .$$

b. Collect terms in the expansion of $\overline{F}(t)$ according to powers of t.

c. Formulate the hazard function using

$$-\log(1-x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots .$$

Applying these steps to the system in Example 4-1 results in the following power series expansion of the system hazard function:

$$R(t) = \lambda_1\lambda_2 t^2 + (\lambda_1\lambda_3\lambda_4 - \tfrac{1}{2}\lambda_1\lambda_2(\lambda_1+\lambda_2))t^3 + (\text{terms of order } t^4).$$

Consequently, the second-order approximation for the hazard function of this system is

$$R^2(t) = \lambda_1\lambda_2 t^2 + (\lambda_1\lambda_3\lambda_4 - \tfrac{1}{2}\lambda_1\lambda_2(\lambda_1+\lambda_2))t^3 .$$

Note that the second-order approximation in this example includes the first-order and the fix-up approximations, i.e.,

$$\begin{aligned} R^2(t) &= \lambda_1\lambda_2 t^2 + \lambda_1\lambda_3\lambda_4 t^3 - \tfrac{1}{2}\lambda_1\lambda_2(\lambda_1+\lambda_2)t^3 \\ &= R^1(t) + \lambda_1\lambda_3\lambda_4 t^3 - \tfrac{1}{2}\lambda_1\lambda_2(\lambda_1+\lambda_2)t^3 \\ &= R^F(t) - \tfrac{1}{2}\lambda_1\lambda_2(\lambda_1+\lambda_2)t^3 . \end{aligned}$$

The direct approach is a rather tedious method for calculating the second-order approximation. In this section we develop a more efficient method. The development of this method uses the following line of reasoning:

(1) We know that

$$R(t) = a_m t^m + a_{m+1} t^{m+1} + (\text{terms of order } t^{m+2}) .$$

Suppose $\overline{F}(t)$ could be expressed as a power series of the form

$$\overline{F}(t) = 1 - b_m t^m - b_{m+1} t^{m+1} + (\text{terms of order } t^{m+2}).$$

Since $m \geq 2$ (we do not consider simple series systems), we would have,

$$-\log \overline{F}(t) = -\log(1-b_m t^m - b_{m+1} t^{m+1} + (\text{terms of order } t^{m+2}))$$

$$= b_m t^m + b_{m+1} t^{m+1} + (\text{terms of order } t^{m+2}).$$

But, $-\log \overline{F}(t) = R(t)$, so corresponding terms of the power series expansion must be equal, i.e.,

$$a_m = b_m ,$$

$$a_{m+1} = b_{m+1} .$$

Consequently, we will have the desired second-order approximation for the system hazard function if we can express $\overline{F}(t)$ in the form

$$\overline{F}(t) = 1 - a_m t^m - a_{m+1} t^{m+1} + (\text{terms of order } t^{m+2}).$$

(2) We know that

$$\overline{F}(t) = 1 - \Pr(\text{system has failed}),$$

and $\Pr(\text{system has failed}) = \sum_{x \varepsilon K} p(x)$.

So we would hope that $\sum_{x \varepsilon K} p(x)$ can be expressed in the form

$$\sum_{x \varepsilon K} p(x) = a_m t^m + a_{m+1} t^{m+1} + (\text{terms of order } t^{m+2}).$$

Such an expression is possible if and only if for all cuts x in K there exist functions (of the component failure rates) $d_m(x)$ and $d_{m+1}(x)$ such that

$$p(x) = d_m(x) t^m + d_{m+1}(x) t^{m+1} + (\text{terms of order } t^{m+2}) .$$

If for every cut x, p(x) can be expressed in the above form, then

$$F(t) = 1 - \sum_{x \varepsilon K} p(x)$$

$$= 1 - \sum_{x \varepsilon K} d_m(x)t^m - \sum_{x \varepsilon K} d_{m+1}(x)t^{m+1}$$

$$+ \text{(terms of order } t^{m+2}\text{)},$$

and $R^2(t) = \sum_{x \varepsilon K} d_m(x)t^m + \sum_{x \varepsilon K} d_{m+1}(x)t^{m+1}$.

Thus we see that the problem of finding a second-order approximation for the system hazard function can be solved by finding a second-order approximation for p(x) for every cut x, i.e.,

$$p(x) = d_m(x)t^m + d_{m+1}(x)t^{m+1} + \text{(terms of order } t^{m+2}\text{)}$$

$$\doteq d_m(x)t^m + d_{m+1}(x)t^{m+1} .$$

(3) Under the assumptions that component failure rates are constant and component lifetimes are independent we have for any cut x:

$$p(x) = \Pr(\text{system in condition } x)$$

$$= \prod_{i \not\in 0(x)} e^{-\lambda_i t} \prod_{i \varepsilon 0(x)} (1-e^{-\lambda_i t})$$

The first product in p(x) can be expanded as follows:

$$\prod_{i \not\in 0(x)} e^{-\lambda_i t} = \prod_{i \not\in 0(x)} (1 - \lambda_i t = \ldots)$$

$$= 1 - t \sum_{i \not\in 0(x)} \lambda_i + \text{(terms of order } t^2\text{)}.$$

The second product in p(x) can be expanded as follows:

$$\prod_{i\varepsilon 0(x)} (1-e^{-\lambda_i t}) = \prod_{i\varepsilon 0(x)} (\lambda_i t - \frac{1}{2}(\lambda_i t)^2 + \ldots)$$

$$= \prod_{i\varepsilon 0(x)} \lambda_i t - \frac{1}{2}(\prod_{i\varepsilon 0(x)} \lambda_i t)(\sum_{i\varepsilon 0(x)} \lambda_i t) + \ldots$$

$$= t^s \prod_{i\varepsilon 0(x)} \lambda_i \, [1-\frac{1}{2} \sum_{i\varepsilon 0(x)} \lambda_i t$$

$$+(\text{terms of order } t^2)]$$

where s = the size of the cut x.

Multiplying these expansions of the products in p(x), we have:

$$p(x) = [1-t \sum_{i\not\varepsilon 0(x)} \lambda_i + \ldots]\cdot t^s \prod_{i\varepsilon 0(x)} \lambda_i [1-\frac{1}{2}t \sum_{i\varepsilon 0(x)} \lambda_i + \ldots]$$

$$= t^s \prod_{i\varepsilon 0(x)} \lambda_i [1 - (\frac{1}{2} \sum_{i\varepsilon 0(x)} \lambda_i + \sum_{i\not\varepsilon 0(x)} \lambda_i)t$$

$$+(\text{terms of order } t^2)]$$

Thus, for any cut x size s we have

$$p(x) = d_s(x)t^s + d_{s+1}(x)t^{s+1} + (\text{terms of order } t^{s+2})$$

where $d_s(x) = \prod_{i\varepsilon 0(x)} \lambda_i$

and $d_{s+1}(x) = - \prod_{i\varepsilon 0(x)} \lambda_i \, [\frac{1}{2} \sum_{i\varepsilon 0(x)} \lambda_i + \sum_{i\not\varepsilon 0(x)} \lambda_i]$.

Summing p(x) over all cuts according to size we have:

$$\sum_{x\varepsilon K} p(x) = \sum_{x\varepsilon K_m} [d_m(x)t^m + d_{m+1}(x)t^{m+1} + (\text{terms of order } t^{m+2})]$$

$$+ \sum_{x\varepsilon K_{m+1}} [d_{m+1}(x)t^{m+1} + (\text{terms of order } t^{m+2})]$$

$$+ \sum_{\substack{x\varepsilon K_j \\ j\geq m+2}} (\text{terms of order } t^{m+2})$$

$$= \sum_{x\varepsilon K_m} d_m(x)t^m + [\sum_{x\varepsilon K_m} d_{m+1}(x) + \sum_{x\varepsilon K_{m+1}} d_{m+1}(x)]t^{m+1}$$

$$+ (\text{terms of order } t^{m+2})$$

(4) As was shown in (2), the above expansion of $\sum_{x\varepsilon K} p(x)$ leads immediately to the second-order approximation for the hazard function, i.e.,

$$R^2(t) = \sum_{x\varepsilon K_m} d_m(x)t^m + [\sum_{x\varepsilon K_m} d_{m+1}(x) + \sum_{x\varepsilon K_{m+1}} d_{m+1}(x)] \; t^{m+1}$$

where

$$\sum_{x\varepsilon K_m} d_m(x)t^m = t^m \sum_{x\varepsilon K_m} \prod_{i\varepsilon 0(x)} \lambda_i \quad ,$$

$$\sum_{x\varepsilon K_m} d_{m+1}(x) = \sum_{x\varepsilon K_m} - \prod_{i\varepsilon 0(x)} \lambda_i \; [\tfrac{1}{2} \sum_{i\varepsilon 0(x)} \lambda_i + \sum_{i\not\varepsilon 0(x)} \lambda_i]$$

$$\sum_{x\varepsilon K_{m+1}} d_{m+1}(x) = \sum_{x\varepsilon K_{m+1}} \prod_{i\varepsilon 0(x)} \lambda_i$$

This form of the second-order approximation has the following properties:

a. Only cuts of size m and size m+1 need be considered.

b. The first term of the second-order approximation is the first-order approximation, i.e.,

$$\sum_{x \varepsilon K_m} d_m(x)t^m = R^1(t).$$

c. It is equivalent to the approximation that would result from using the direct approach.

The following example illustrates the calculation of the second-order approximation and points out a computational shortcut.

Example 4-2. Calculate the second-order approximation for the system described in Example 4-1.

(a) System width is $m = 2$.

(b) There is one cut of size 2: $x^1 = (0,0,1,1)$.

So, $K_2 = \{x^1\}$.

(c) There are three cuts of size 3:

$x^2 = (0,1,0,0)$,

$x^3 = (0,0,1,0)$,

$x^4 = (0,0,0,1)$.

So, $K_3 = \{x^2,x^3,x^4\}$.

(Note that x^2 is a minimal cut but x^3 and x^4 are not minimal cuts.)

(d) Calculate $\sum_{x \varepsilon K_m} d_m(x)t^m$.

$$\sum_{x \varepsilon K_2} d_2(x)t^2 = d_2(x^1)t^2 = t^2 \prod_{i \varepsilon 0(x^1)} \lambda_i = \lambda_1\lambda_2 t^2 .$$

(e) Calculate $\sum_{x \varepsilon K_m} d_{m+1}(x)t^{m+1}$. We will carry out this calculation in two parts:

$$\sum_{x \varepsilon K_2} - \prod_{i \varepsilon 0(x)} \lambda_i \left[\frac{1}{2} \sum_{i \varepsilon 0(x)} \lambda_i\right] t^3 = -\frac{1}{2}\lambda_1\lambda_2(\lambda_1 + \lambda_2)t^3 .$$

$$\sum_{x\varepsilon K_2} - \prod_{i\varepsilon 0(x)} \lambda_i \left[\sum_{i\not\varepsilon 0(x)} \lambda_i \right] t^3 = - \lambda_1\lambda_2\lambda_3 t^3 - \lambda_1\lambda_2\lambda_4 t^3 .$$

(The latter pair of terms will be cancelled when we consider the cuts in K_{m+1} which are not minimal cuts.)

(f) Calculate $\sum_{x\varepsilon K_{m+1}} d_{m+1}(x) t^{m+1}$.

$$d_3(x^2)t^3 = \lambda_1\lambda_3\lambda_4 t^3 ,$$

$$d_3(x^3)t^3 = \lambda_1\lambda_2\lambda_4 t^3 ,$$

$$d_3(x^4)t^3 = \lambda_1\lambda_2\lambda_3 t^3 .$$

(The last two terms will cancel terms from (e).)

(g) Sum the results of (d), (e), and (f).

$$R^2(t) = \lambda_1\lambda_2 t^2 - \frac{1}{2}\lambda_1\lambda_2(\lambda_1+\lambda_2)t^3 + \lambda_1\lambda_3\lambda_4 t^3 .$$

In the foregoing example the terms $-d_3(x^3)t^3$ and $-d_3(x^4)t^3$ appear in the term $d_3(x^1)t^3$. This is a consequence of the fact that x^1 is contained in both x^3 and x^4, i.e., $0(x^1)$ is a proper subset of both $0(x^3)$ and $0(x^4)$. We formalize this result in the following manner:

For each cut x^* of size m+1, let $v(x^*)$ be defined as the number of cuts of size m which are contained in x^*. The term $d_{m+1}(x^*)t^{m+1}$ will appear in $\sum_{x\varepsilon K_m} d_{m+1}(x)t^{m+1}$ with the coefficient $-v(x^*)$. Consequently, the term $d_{m+1}(x^*)t^{m+1}$ will appear in $R^2(t)$ with the coefficient $1-v(x^*)$, and this is true for each cut x^* of size m+1. Thus we may rewrite $R^2(t)$ in the form

$$R^2(t) = \sum_{x \varepsilon K_m} d_m(x) t^m - \frac{1}{2} \sum_{x \varepsilon K_m} d_m(x) \left[\sum_{i \varepsilon 0(x)} \lambda_i \right] t^{m+1}$$

$$+ \sum_{x \varepsilon K_{m+1}} (1-v(x)) d_{m+1}(x) t^{m+1} .$$

Thus, the second-order approximation for the system hazard function can be calculated using the following method:

(a) Determine the system width m.

(b) Identify each cut of size m.

(c) For each cut of size m compute:

(1) $d_m(x) = \prod_{i \varepsilon 0(x)} \lambda_i$,

(2) $\sum_{i \varepsilon 0(x)} \lambda_i$.

(d) Identify each minimal cut of size m+1, and compute $d_{m+1}(x) = \prod_{i \varepsilon 0(x)} \lambda_i$. (Note that $v(x) = 0$ for minimal cuts.)

(e) Every non-minimal cut of size m+1 contains at least one cut of size m, i.e., $v(x) \geq 1$. Identify those non-minimal cuts of size m+1 which contain more than one cut of size m, and compute $(1-v(x))d_{m+1}(x)$ for each of these. (If we miss some terms here, the error is usually small and always on the "safe side.")

(f) Use the terms calculated in (c), (d) and (e) to calculate $R^2(t)$.

The following example illustrates the method.

Example 4-3. In the system discussed in Example 4-1 place another pump, No. 5, in series with No. 1. The resulting system is shown in Fig. 5.

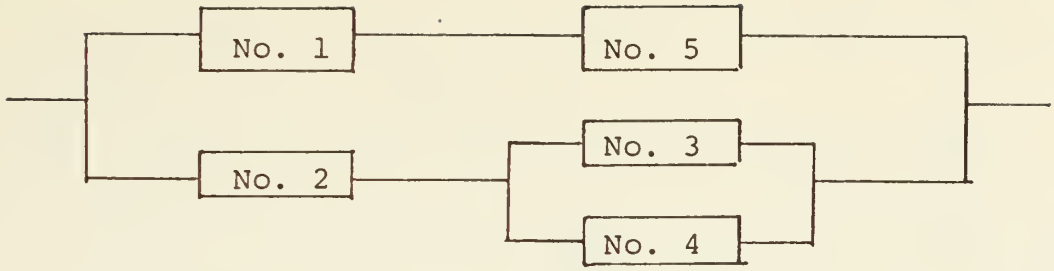

Figure 5. Pump System - Physical Arrangement.

The second-order approximation for the system hazard function is calculated as follows:

(a) At least two pumps must fail in order for the system to fail, so $m = 2$.

(b) The cuts of size 2 are:

$x^1 = (0,0,1,1,1)$ (Nos. 1 and 2 have failed)

$x^2 = (1,0,1,1,0)$ (Nos. 2 and 5 have failed)

(c) $d_2(x^1) = \lambda_1\lambda_2$ $\qquad \sum_{i\varepsilon O(x^1)} = \lambda_1 + \lambda_2$

$d_2(x^2) = \lambda_2\lambda_5$ $\qquad \sum_{i\varepsilon O(x^2)} \lambda_i = \lambda_2 + \lambda_5$

(d) There are two minimal cuts of size $m+1 = 3$:

$x^3 = (0,1,0,0,1)$ (Nos. 1, 3 and 4 have failed)

$x^4 = (1,1,0,0,0)$ (Nos. 3, 4 and 5 have failed).

$d_3^3(x^3) = \lambda_1\lambda_3\lambda_4$

$d_3(x^4) = \lambda_3\lambda_4\lambda_5$

(e) The cut $x^5 = (0,0,1,1,0)$ contains both x^1 and x^2, i.e., $O(x^1) = \{1, 2\}$

$O(x^2) = \{2, 5\}$

and both of these are proper subsets of $O(x^5) = \{1, 2, 5\}$. So, $v(x^5) = 2$,

and $(1-v(x^5))d_3(x^5) = -\lambda_1\lambda_2\lambda_5$.

All other non-minimal cuts of size 3 contain only one cut of size 2, i.e., $1-v(x) = 0$.

(f) Using the terms from (c), (d) and (e) we have

$$R^2(t) = (\lambda_1\lambda_2+\lambda_2\lambda_5)t^2 - \frac{1}{2}[\lambda_1\lambda_2(\lambda_1+\lambda_2) + \lambda_2\lambda_5(\lambda_2+\lambda_5)]t^3 + (\lambda_1\lambda_3\lambda_4+\lambda_3\lambda_4\lambda_5-\lambda_1\lambda_2\lambda_5)t^3 .$$

The most difficult task in formulating the second-order approximation is the identification of the non-minimal cuts of size m+1 which contain more than one cut of size m. Aside from this difficulty the second-order approximation is conceptually no more difficult than the fix-up approximation. Hand calculating the second-order approximation may be relatively tedious. Whether increased accuracy is worth extra effort is a difficult question in almost any context; we only note that in many cases the error of the second-order approximation is less than one-tenth the error of the fix-up approximation.

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13. ABSTRACT

Methods for approximating the system hazard function are developed for systems which have constant component failure rates. The approximations are applicable to systems which are "highly reliable," e.g., all component reliabilities greater than 0.9 and system reliability greater than 0.95.

Three approximations are developed. The first-order approximation is based on the system cuts of least size (smallest cuts). The fix-up approximation is based on the minimal cuts of the system and is an extension of the methods used in NAVWEPS OD 29304. The second-order approximation is a more accurate extension of the first-order approximation.

The advantages peculiar to each of these approximations are:

(1) first order: easy to calculate,
(2) fix-up: never leads to an overestimate of system reliability,
(3) second-order: relatively more accurate.

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